CS 357 Transformations

The View Transformation

$$V = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -a_{v} & -b_{v} & -c_{v} & 1 \end{bmatrix} \times \begin{bmatrix} a_{x} / & -b_{x} / & 0 & 0 \\ b_{x} / & a_{x} / & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} r_{R} & 0 & -c_{x} / & 0 \\ 0 & 1 & 0 & 0 \\ c_{x} / R & 0 & r_{R} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ \times \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & c_{z} R / & b_{z} a_{x} - a_{z} b_{x} & 0 \\ 0 & c_{z} R / & b_{z} a_{x} - a_{z} b_{x} & 0 \\ 0 & b & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \pm 1 & 0 & 0 \\ 0 & \pm 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where
$$r = \sqrt{a_x^2 + b_x^2}$$
, $R = \sqrt{a_x^2 + b_x^2 + c_x^2}$, and $h = r\sqrt{a_z^2 + b_z^2 + c_z^2}$

The Perspective Transformation

$$P = \begin{bmatrix} D & 0 & 0 & 0 \\ 0 & D & 0 & 0 \\ 0 & 0 & \frac{Y}{Y - H} & 1 \\ 0 & 0 & \frac{-HY}{Y - H} & 0 \end{bmatrix}$$

where H, D, and Y are distances to the Hither plane, viewplane and Yon plane.

The Window Transformation

$$W = \begin{bmatrix} \frac{W_R - W_L}{v_R - v_L} & 0 & 0 & 0\\ 0 & \frac{W_T - W_B}{v_T - v_B} & 0 & 0\\ 0 & 0 & 1 & 0\\ \frac{W_L v_R - v_L W_R}{v_R - v_L} & \frac{W_B v_T - v_B W_T}{v_T - v_B} & 0 & 1 \end{bmatrix}$$

where the v's and w's are respectively the boundaries of the viewport on the viewplane and the window on the screen.

The full view pipeline, which transforms object coordinates into screen coordinates, is [h, v, z, 1] = [x, y, z, 1]VPW < homogenize . >= [x, y, z, 1]VP < homogenize > W

Example: Suppose the viewer is at (10, 10, 10) looking at point (0,0,0), with the viewer's up-vector <0,0,1>. The viewplane is z=10 (i.e, D =10); the hither and yon planes are defined by Y=15 and H = 5. We use a view angle of Θ =40 degrees, and use window boundaries 0 to 500 in each direction. Where on the screen is the point (2,5, 1) (specified in world coordinates) drawn?

$$\begin{aligned} (a_v, b_v, c_v) &= (10, 10, 10) & \langle a_z, b_z, c_z \rangle = \langle -10, -10, -10 \rangle \\ \langle a_x, b_x, c_x \rangle &= \langle a_z, b_z, c_z \rangle x \ \mathbf{up} = \langle b_z, -a_z, 0 \rangle = \langle -10, 10, 0 \rangle \\ r &= \sqrt{200} = R \quad h = r \sqrt{a_z^2 + b_z^2 + c_z^2} = 100\sqrt{6} \quad \text{This is enough to find V:} \\ V &= \begin{bmatrix} -0.707 & -0.408 & -0.578 & 0 \\ 0.707 & -0.408 & -0.578 & 0 \\ 0 & 0.818 & -0.578 & 0 \\ 0 & 0 & 17.33 & 1 \end{bmatrix} \end{aligned}$$

We are given that $D = 10$, $V = 15$ and $H = 5$, so $D = \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 & 0 \end{bmatrix}$

We are given that D = 10, Y = 15 and H = 5, so $P = \begin{bmatrix} 0 & 10 & 0 & 0 \\ 0 & 0 & 1.5 & 1 \\ 0 & 0 & -7.5 & 0 \end{bmatrix}$

 $v_{L} = -D \tan(40) = -8.39 \text{ Similarly, } v_{R} = v_{T} = 8.39, v_{B} = -8.39.$ The window boundaries are given as $w_{L}=0, w_{R} = 500, w_{T} = 0, w_{B} = 500.$ This gives $W = \begin{bmatrix} 29.79 & 0 & 0 & 0 \\ 0 & -29.79 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 250 & 250 & 0 & 1 \end{bmatrix}$

 $\begin{bmatrix} 250 & 250 & 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 2 & 5 & 1 & 1 \end{bmatrix} VPW = \begin{bmatrix} 2.12 & -2.04 & 12.71 & 1 \end{bmatrix} PW = \begin{bmatrix} 21.2 & -20.4 & 11.56 & 12.71 \end{bmatrix} W$ $= \begin{bmatrix} 3808.35 & 3783.62 & 11.56 & 12.71 \end{bmatrix}$

This last result homogenizes to [299.7. 297.8, 0.91, 1]. Note that the first two coordinates are within our window boundaries and the z-coordinate is between 0 and 1. We would draw the point at pixel (300, 298).